Math 1545 Calculus II — Prof. Paul Bailey — Worksheet 1 — February 21, 2006 — Name:

Volumes, Areas, and Lengths

Due Friday, March 3, 2006. Attach this sheet to your solutions.

Volume by Disks: $V = \int_a^b \pi r^2 \, du$

Volume by Washers: $V = \int_a^b \pi (R^2 - r^2) \, du$

Volume by Shells: $V = \int_a^b 2\pi r h \, du$

Length: $L = \int_a^b ds$

Area: $A = \int_a^b 2\pi r \, ds$

Parametric Differential: $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

Functional Differential: $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

Example 1. Let D be the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and the x-axis. Find the volume generated by revolving D around the x-axis.

Solution. Since we are revolving around the x-axis, it is convenient to take vertical slices, one for each value of x. The slices are then disks, and we use the disk method. The radius is $r = y$, and we solve for y to find that $y = 9(1 - \frac{x^2}{4})$. The limits of integration are found by intersecting the ellipse and the line $y = 0$. This gives $a = -2$ and $b = 2$. Thus

$$\begin{aligned} V &= \int_{-2}^{2} \pi y^2 \, dx \\ &= 2 \int_0^2 \pi (9) \left(1 - \frac{x^2}{4}\right) dx \quad \text{because we integrate an even function} \\ &= 18\pi \left[x - \frac{1}{4} \cdot \frac{x^3}{3}\right]_0^2 \\ &= 18\pi \left[2 - \frac{8}{12}\right] \\ &= 24\pi. \end{aligned}$$

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Problem 1. Let D be the region in the upper half plane bounded by the parabola $y = 25 - x^2$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Find the volume generated by revolving D around the x-axis.

Problem 2. Let D be the region in the upper half plane bounded by the parabola $y = 25 - x^2$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Find the volume generated by revolving D around the line $y = 10$.

Problem 3. Let D be the region in the right half plane bounded by the ellipse $\frac{x^2}{16} + y^2 = 1$ and the line $x = 0$. Find the area of the surface generated by revolving D around the y-axis.

Problem 4. Let $f(x) = \frac{x^3}{12} + \frac{1}{x}$. Find the length of the graph of f between $x = 1$ and $x = 2$.

Problem 5. The *Archimedean spiral* is parameterized by $\gamma(t) = (t\cos t, t\sin t)$. Find the arclength of this curve from $t = 0$ to $t = 2\pi$.